Impairment Estimates of Equity Portfolios Represented by Model Points

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Abstract.. Equity instruments in simulation-based risk models are usually represented by model points in an aggregated view. Each model point represents all equity instruments of a sub portfolio (e.g., all equity instruments in a certain currency, a certain region or asset category). This approach is sufficient for the simulation of economic figures like average return of the portfolio and its correlation with other quantities. Accounting figures like impairments, however, need to be determined on a single asset level. For each simulation scenario, an average return of the portfolio is provided (usually by an Economic Scenario Generator). Based on this average return, scenario specific impairments have to be derived. The whole simulation consists of many (at least several thousand) scenarios therefore a reliable and fast procedure for the impairment calculation is needed. In this paper, we present a statistical approach for this purpose and compare its performance against a real portfolio of shares.

Keywords: Solvency II, Internal Models, Dynamic Financial Analysis (DFA), Equity Impairments, Economic Scenario Generator

1 INTRODUCTION

Actuarial modeling (MacDonald, 2009) is gaining importance in the insurance sector, especially in the context of Solvency II (as adopted by the European Parliament and the Council, 2009). Assets and liabilities are simulated in a set of scenarios usually generated by an Economic Scenario Generator. As a result, the development of a company's economic situation under different conditions can be evaluated.

Part of the outcome of such models is the profit and loss of the insurance company for each of the simulated scenarios. This requires the application of accounting rules to the economic figures generated in the scenarios. Of special interest in this context are impairment losses on equity instruments as part of the company's assets: Under IFRS accounting rules (IASB, 2009), insurers usually classify a decline in the fair value of a specific equity instrument as profit and loss once it falls below a certain threshold. This single-asset view does not comply with representing several equity instruments by one model point in the simulation.

Batens (2009) has proposed to use statistical estimates to overcome this difficulty based on a log-normal distribution of equity returns where the portfolio return remains a stochastic variable. In this paper, we make use of the general feature of actuarial models to provide an average return in each scenario. Consequently, we obtain a conditional distribution (given the average return) for the return of each equity instrument which is then used to derive scenario-specific impairment losses. In addition, assuming an intuitive relationship between the real portfolio and its representation by a model point, we compare the corresponding impairment loss estimates and find the agreement to be satisfactory.

As a basis for the statistical estimates, we approximate returns by a normal distribution for the sake of simplicity and generality. Log-normal returns could easily be incorporated by an appropriate transformation of parameters (Bühlmann, 2005). The estimation procedure can also be applied to other families of distributions where additional numerical effort might be required when closed-form solutions are not available.

2 DESCRIPTION OF THE PROBLEM

2.1 Simulated and desired quantities

Starting from a risk model implemented by simulations, each scenario is characterized by a linear (price) return $\bar{R}$ of the portfolio; where in a typical simulation several thousand scenarios will be generated, each with a different $\bar{R}$. If S_0 is the initial value of the portfolio, the simulated final value $\bar{R}_1$ in each scenario is given as

$$\bar{S}_1 = \left(1+\bar{R}\right)S_0 \, .$$

We would like to estimate the (average) impairment loss in each scenario. Assuming that the portfolio consists of equity instruments classified as available for sale according to IAS 39 (IASB, 2009), gains or losses for

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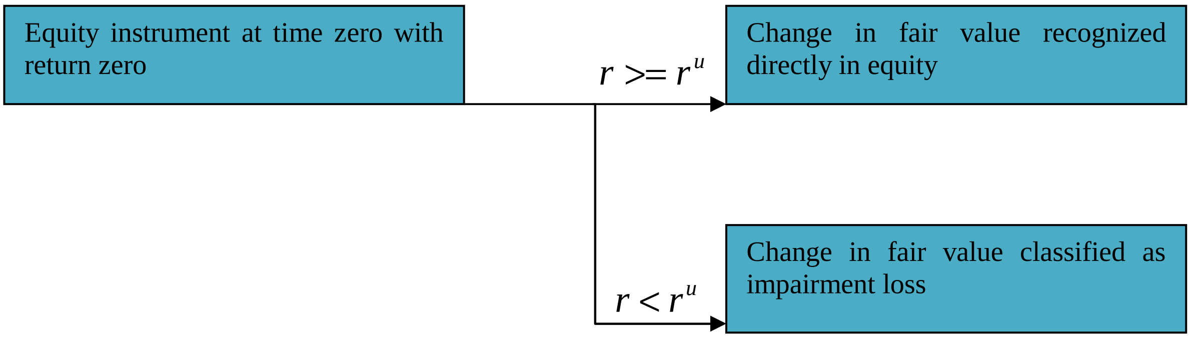

Figure 1: Impact of changes in fair value on balance sheet

each equity instrument are generally recognized directly in equity until the asset is sold. According to IAS 39.67, if there is objective evidence for significant decline of the fair value, a corresponding amount needs to be reclassified as profit and loss for the respective period[2]. Significant decline can be interpreted as a drop in fair value below a certain threshold w.r.t. the amortized costs (acquisition costs). In our model of linear returns, impairment will occur if the return of the equity instrument is below a threshold $r^u < 0$. The impairment loss consists of the difference between the acquisition costs and the current fair value.

As a consequence, if r is the return of the equity instrument, the change in the fair value is entered in the balance sheet as depicted in figure 1.

2.2 Equity and model portfolio

In the remaining part of this paper, we assume that the (real) equity portfolio consists of n different equity instruments with equal weights[3]. Their individual returns r_i are assumed to be multivariate normal distributed with parameters $\underline{\mu} = (\mu_1,\ldots,\mu_n)$ and $\sigma_{ij} = \rho_{ij}\sigma_i\sigma_j$. The simulation, however, is not based on these parameters. Instead, the portfolio is represented by a model point of n equal multivariate normal distributed equity instruments with return μ_{MP}, volatililty σ_{MP} and correlation ρ_{MP}[4]. Instead of individual asset returns, only the average return

$$R = \frac{1}{n}\sum_{i=1}^{n} r_i$$

is simulated: It is set to a value $\bar{R}$ in each scenario.

To compare the estimates of the impairment losses for the real and the model point portfolio, we require both portfolios to have the same mean and variance. Correlation of the model point portfolio is obtained by averaging, resulting in

$$\mu_{MP} = \frac{1}{n}\sum_{i=1}^{n}\mu_i ,$$

$$\sigma_{MP} = \sqrt{\frac{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}\sigma_i\sigma_j}{\sum_{i=1}^{n}\sum_{j=1}^{n}\rho_{ij}}} ,$$

$$\rho_{MP} = \frac{1}{(n-1)n/2}\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\rho_{ij} .$$

Note, that since we only have one condition, i.e. same portfolio variance, to fix model point variance and correlation, other choices for these parameters are possible. However, we believe that our choice is the most intuitive.

To obtain a (positive semidefinite) correlation matrix, we need

$$\rho_{MP} \geq -\frac{1}{n-1}$$

which can be derived from the positive semidefiniteness of $(\rho_{ij})\sigma_{MP}$ is well defined as long as (ρ_{ij}) is strictly positive definite. Thus we only have to exclude maximal anticorrelation in the real portfolio (to be numerically stable, also nearly maximal anticorrelation).

2 Impairments due to prolonged decline in the fair value are not considered here.
3 Generalization to different weights is straightforward.
4 The portfolio might be modelled with a different type of distribution in the simulation; however, parameters should be as consistent as possible.

3 STATISTICAL ESTIMATES

3.1 Analytical Description

By fixing the average return to a value $\overline{R}$ in each scenario, the return of each equity instrument is determined by a conditional distribution which is again a normal distribution with different parameters (Flury, 1997). The derivation of the following formulas can be found in the appendix.

For the model portfolio, the conditional distribution turns out to have a mean equal to the average return,

$$E_{MP}(r_i \mid R = \overline{R}) = \overline{R} \quad ,$$

and the variance is reduced to

$$Var_{MP}(r_i \mid R = \overline{R}) = \sigma^2{}_{MP}\left(\frac{n-1}{n}(1-\rho_{MP})\right) \quad .$$

For the real portfolio, the parameters are

$$E_{real}(r_i \mid R = \overline{R}) = \mu_i + \frac{n\sigma_i \sum_{j=1}^{n} \rho_{ij}\sigma_j}{\sum_{j,k=1}^{n} \rho_{jk}\sigma_j\sigma_k}\left(\overline{R} - \frac{1}{n}\sum_{j=1}^{n}\mu_j\right)$$

and

$$Var_{real}(r_i \mid R = \overline{R}) = \sigma_i^{\,2}\left(1 - \frac{\left(\sum_{j=1}^{n}\rho_{ij}\sigma_j\right)^2}{\sum_{j,k=1}^{n}\rho_{jk}\sigma_j\sigma_k}\right) \quad .$$

Starting with zero return at time zero, the impairment probability (or fraction of impaired equity instruments) $p_{i,A}$ and expected impairment loss A_i for each equity instrument i in the scenario with average return $\overline{R}$ is determined by the cumulative distribution and expected shortfall, respectively, w.r.t. r^u:

$$p_{i,A} = P(r_i < r^u \mid R = \overline{R}) = \Phi(\tilde{r}^u)$$

$$\begin{aligned} A_i &= p_{i,A} E(r_i \mid r_i < r^u, R = \overline{R}) \\ &= \Phi(\tilde{r}^u) E(r_i \mid R = \overline{R}) - \phi(\tilde{r}^u)\sqrt{Var(r_i \mid R = \overline{R})} \end{aligned}$$

where $\Phi(x)$ and $\phi(x)$ are the cumulative distribution function and probability density function, respectively, of the standard normal distribution and $\tilde{r}^u$ the centralized threshold for impairments:

$$\tilde{r}^u = \frac{r^u - E(r_i \mid R = \overline{R})}{\sqrt{Var(r_i \mid R = \overline{R})}} \quad .$$

If the equity instruments have a nonzero initial return or are already impaired, the estimates can be easily adapted. Likewise, a multi-period treatment can be implemented.

3.2 Effect of correlation

As a function of the correlation ρ, the fraction of impaired equity instruments and expected impairment losses are shown for an average return of $\overline{R} = 5\%$ (above the impairment threshold) and $\overline{R} = -15\%$ (below the impairment threshold) in figures 2 and 3 respectively.

With increasing correlation, the conditional variance of the equity instruments is reduced. When the equity instruments are perfectly correlated, their return is shrunk to a single value (the average return $\overline{R}$), resulting in a conditional variance of zero. In this case, either all or none of the equity instruments are impaired. The fraction of impaired equity instruments is approaching these limiting values when correlation is increased. Consequently, in this limit, the impaired losses also tend to zero when the average return is above the impairment threshold. For an average return below the impairment threshold, two effects counterbalance each other: When correlation is increased, more equity instruments are impaired, but the impairment loss for each of these is reduced due to the absence of large negative returns. When averaging over all instruments (impaired and not impaired), the impairment losses are nearly independent of correlation in this case and close to their limiting value of $\overline{R}$ for perfect correlation.

4 EXAMPLE: DOW JONES INDUSTRIAL AVERAGE

To get an impression of the validity of the estimates, we compare the fraction of impaired equity instruments and impairments losses for the Dow Jones Industrial Average (which is composed of thirty shares with equal weights). Parameters are taken from Bloomberg as annualized quantities (up to 16.02.2010) with an impairment threshold of $r^u = -20\%$. The results of this comparison are shown in figures 4 and 5.

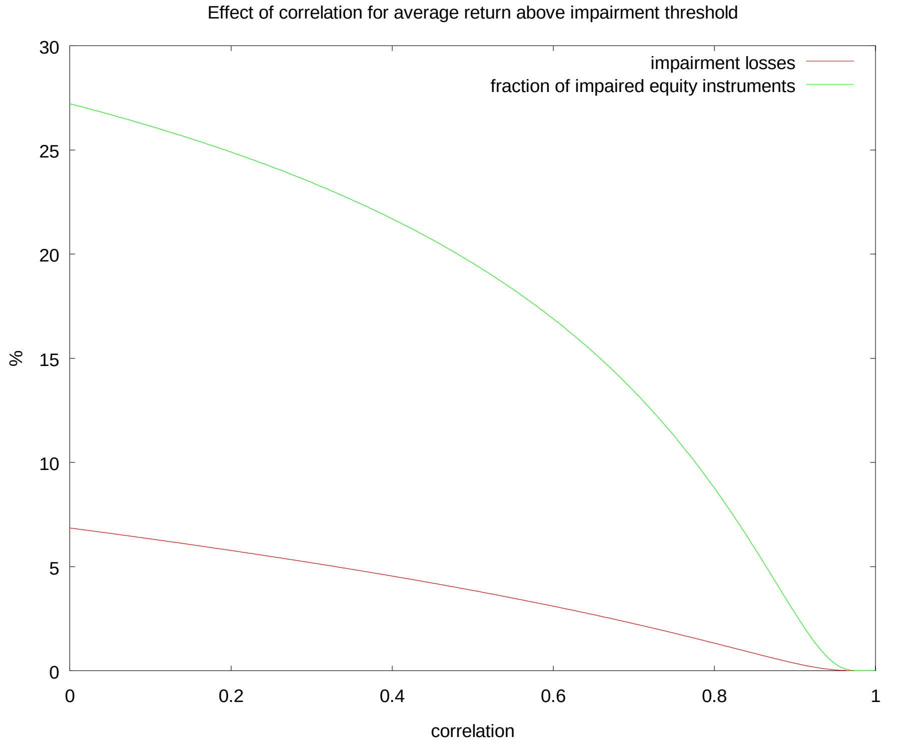

Figure 2: Effect of correlation for average return above impairment threshold

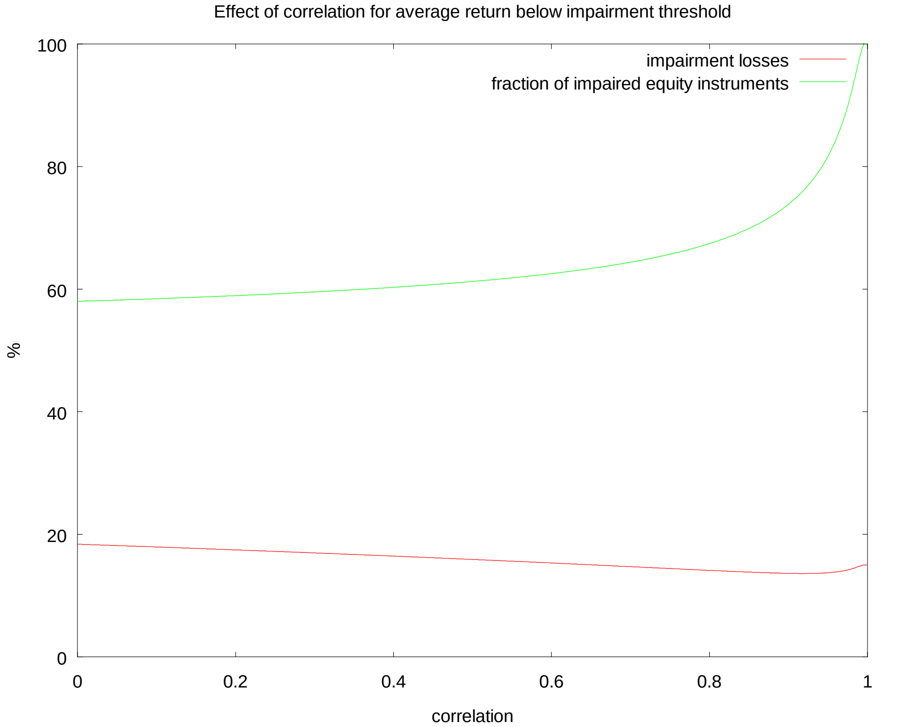

Figure 3: Effect of correlation for average return below impairment threshold

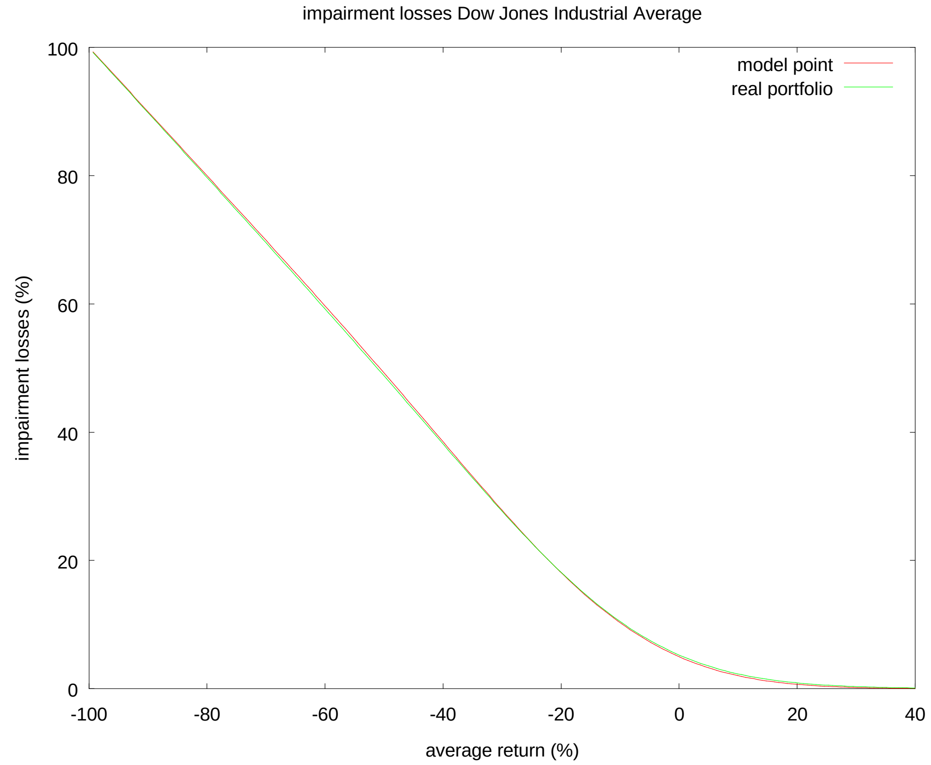

Figure 4: Comparison of simulated against estimated impairment losses

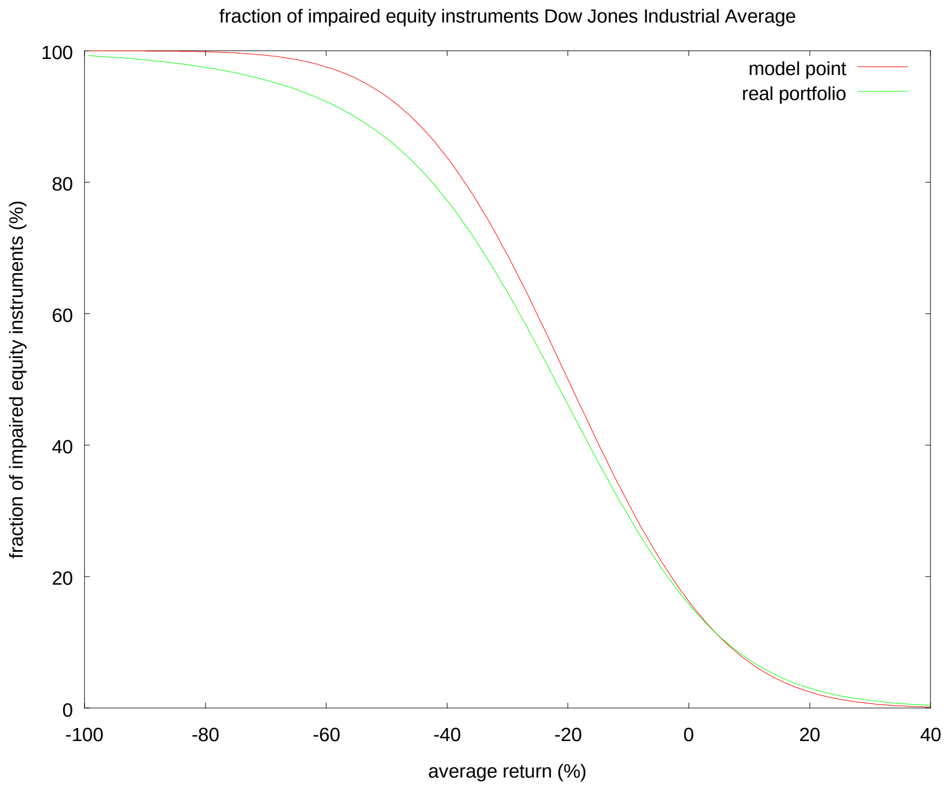

Figure 5: Comparison of fraction of impaired equity instruments

For low average returns, less equity instruments will be impaired in the real than in the model point portfolio. This can be attributed to equity instruments with low volatility and correlations in the real portfolio: Their conditional mean is only slightly influenced by the average return as can be inferred from the formula in section 3.1. The expected impaired losses, however, are very well described by the model point portfolio - the different losses for the individual equity instruments in the real portfolio compensate each other. For an average return clearly below the impairment threshold the expected impairment losses depend linearly on and almost equal the average return.

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APPENDIX

This appendix contains the derivation of the statistical estimates in section 3. To determine the conditional distribution, we start with the general situation of two bivariate normal distributed random variables (X,Y) with mean (μ_X,μ_Y), variance (σ_X^2,σ_Y^2) and correlation ρ_{XY}. According to Flury (1997), the conditional distribution of X given $Y = y$ is again a normal distribution:

$$X \mid Y = y \sim N(\mu_X + \rho_{XY} \frac{\sigma_X}{\sigma_Y}(y - \mu_Y), \sigma^2{}_X(1 - \rho^2{}_{XY})).$$

For the real portfolio, the return R is normal distributed with mean

$$\mu_R = \frac{1}{n}\sum_{i=1}^{n} \mu_i$$

and variance

$$\sigma^2{}_R = \frac{1}{n^2}\sum_{i,j=1}^{n} \rho_{ij}\sigma_i\sigma_j .$$

The common distribution of (r_i,R) is a bivariate normal distribution with covariance

$$\begin{aligned} Cov(r_i, R) &= Cov(r_i, \frac{1}{n}\sum_{j=1}^{n} r_j) \\ &= \frac{1}{n}\sum_{j=1}^{n} Cov(r_i, r_j), \\ &= \frac{1}{n}\sum_{j=1}^{n} \rho_{ij}\sigma_i\sigma_j \end{aligned}$$

i.e. correlation

$$\rho(r_i, R) = \frac{1}{n}\frac{\sum_{j=1}^{n}\rho_{ij}\sigma_i\sigma_j}{\sigma_i\sqrt{\frac{1}{n^2}\sum_{j,k=1}^{n}\rho_{jk}\sigma_j\sigma_k}} = \frac{\sum_{j=1}^{n}\rho_{ij}\sigma_j}{\sqrt{\sum_{j,k=1}^{n}\rho_{jk}\sigma_j\sigma_k}}$$

Consequently, given $R = \bar{R}$, each r_i is normal distributed with mean

$$\mu_i + \frac{\sum_{j=1}^{n}\rho_{ij}\sigma_j}{\sqrt{\sum_{j,k=1}^{n}\rho_{jk}\sigma_j\sigma_k}} \frac{\sigma_i}{\frac{1}{n}\sqrt{\sum_{j,k=1}^{n}\rho_{jk}\sigma_j\sigma_k}}(\bar{R} - \frac{1}{n}\sum_{j=1}^{n}\mu_j)$$

$$= \mu_i + \frac{n\sum_{j=1}^{n}\rho_{ij}\sigma_i\sigma_j}{\sum_{j,k=1}^{n}\rho_{jk}\sigma_j\sigma_k}(\bar{R} - \frac{1}{n}\sum_{j=1}^{n}\mu_j)$$

and variance

$$\sigma^2{}_i(1 - \frac{(\sum_{j=1}^{n}\rho_{ij}\sigma_j)^2}{\sum_{j,k=1}^{n}\rho_{jk}\sigma_j\sigma_k})$$

The corresponding result for the model portfolio can be obtained by setting

- $\mu_j = \mu_{MP}$ ($j = 1,\ldots,n$)
- $\sigma_j = \sigma_{MP}$ ($j = 1,\ldots,n$)
- $\rho_{jk} = \rho_{MP}$ ($j \neq k$, $j,k = 1,\ldots,n$)

($\rho_{jk} = 1$ holds in general)

Note that this is just a way of deriving the conditional distribution of the model point portfolio, not the actual relation between the model point portfolio and the real portfolio.

Finally, to estimate the impairment loss, we need the expected shortfall (conditional expectation for a random variable below (or above) a certain threshold). For a normal distributed random variable X with mean μ and variance σ^2, it is known to be

$$E(X, X < x) = \mu - \sigma \frac{\phi(\tilde{x})}{\Phi(\tilde{x})}$$

with the probability density function ϕ and cumulative distribution function Φ of the standard normal distribution and the centralized threshold

$$\tilde{x} = \frac{x - \mu}{\sigma}.$$